CSCI 5521: Pattern Recognition

Problem set 1:

9/4/03 **Due: 9/18/03 4:00pm**

Download the file arrow.m:
http://www.mathworks.com/matlabcentral/fileexchange/loadFile.do?objectId=278&objectType=file
Use the arrow command to visualize vectors.

1. Consider the vectors $x_1 = [1,0]^T$; $x_2 = [1, \frac{1}{\sqrt{2}}]^T$; Compute the area formed by the parallelpiped of these two vectors using the formula for the area of a right triangle $Area = \frac{1}{2}(base * height)$. Now put the two vectors into a matrix **A**, and compute the determinant. What is the relationship between det(**A**) and area? What if either one or both x_i is negated? Now let
$x_1 = [0.86603, 0.5]^T$; $x_2 = [0.51247, 1.11237]^T$; Again use the right triangle formula and the determinant to determine the area (it may help to visualize using arrow.m) and the det(). Can you find a relationship between these two formulae?
2. Consider the equation $\vec{y} = A\vec{x} \Leftrightarrow \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} b_1^T \\ b_2^T \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$
Let
$b_1 = \begin{bmatrix} -\sqrt{3}/2 \\ \frac{1}{2} \end{bmatrix}$; $b_2 = \begin{bmatrix} \sqrt{3}/2 \\ \frac{1}{2} \end{bmatrix}$; $y = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$
Using Matlab, plot the constraint lines determined by b_1 and b_2 for the range –4:4, i.e. determine the lines perpendicular to b_1 and b_2 such that the dot product between the b_i and any other vector is equal to {-4,-3,-2,…,4}, and display them using the line() command or the plot() command. Graphically solve for x. Use matrix inversion to solve for x.
3. Let x and y be identically Gaussian random variables:
$p(x) = N(\mu, \sigma_x) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma_x^2})$
$p(y \mid x) = N(x, \sigma_y) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp(-\frac{1}{2}\frac{(y-x)^2}{\sigma_y^2})$

   Choose values for μ and both σ. Sample x and y over appropriate ranges and construct probability tables for both distributions to turn the continuous distributions into discrete distributions. Make sure the sum-to-1 constraints are satisfied. Compute the probability tables for *p(y)* and *p(x|y)*.
4. Generate a set of 100 random *x,y* coordinates using rand(). Use scatter() to display them. All the points should live in the unit square. Let *s* denote 1 point. Compute the average inner product of these points: $E[s^T \cdot s] = \frac{1}{100} \sum_{j=1:100} s_j^T \cdot s_j$.

Transform each of these points by a matrix $Q = \begin{bmatrix} 1.43 & -0.48 \\ 1.25 & 0.57 \end{bmatrix}$, so that $z = Q \cdot s$.

Scatterplot z. Compute the average inner product of the *z* points. Use an eigenanalysis of Q to explain the difference between the *s* and the *z* lengths. Compute the area the transformed points live in using Q. Using the relationship $Q=A\Lambda A^{-1}$, (*where A* contains the normalized eigenvectors and Λ is a diagonal matrix of eigenvalues) to determine the transform executed by *A*. Use arrow() to plot the columns of A with the scatterplot of *z*. The net effect of *Q* on the length of the vectors *s* is a rotation and a scaling introduced by Q^TQ. Find the angle of the rotation executed by Q^TQ. Hint: do an eigenanalysis of Q^TQ, make sure the eigenvectors (e.g. *A)* are length 1 and has det of 1, then use the parametric form of a 2-D rotation matrix: $R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$.

5. Compute the distribution of the sum of three fair 6-sided dice.
6. Dice difference: Dice difference is a game that involves two players. Two dice are throw, and the absolute value of the difference computed. Player A wins if the value is {0,1,2}. Player B wins if the value is {3,4,5}. Is the game fair? Justify your answer. Propose modified rules to make the game fair and justify the new rules.