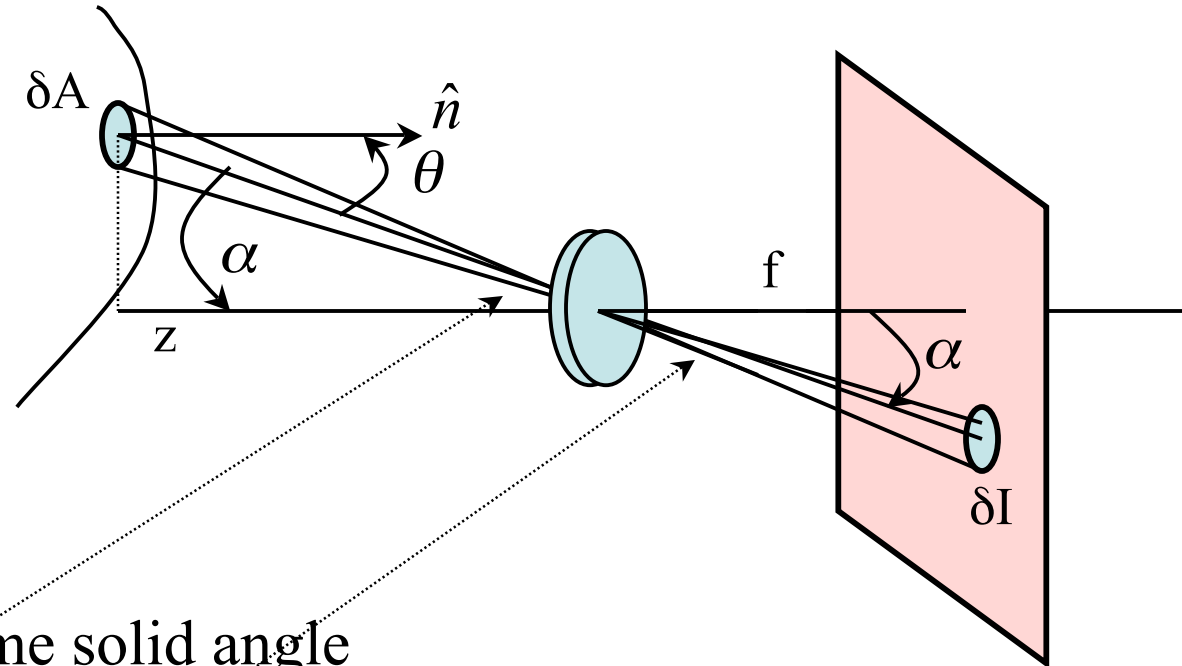


Shading, Shadows, Lights, and shape from shading

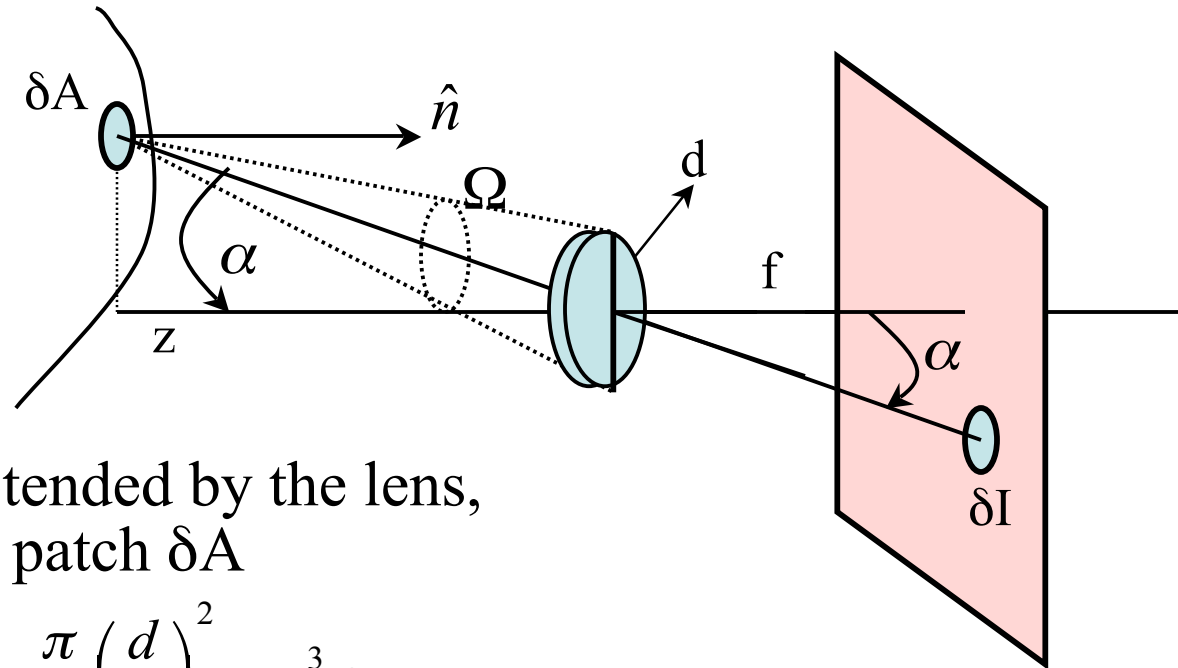
Surface Radiance and Image Irradiance

Pinhole
Camera
Model



$$\frac{\delta A \cos \theta}{(z / \cos \alpha)^2} = \frac{\delta I \cos \alpha}{(f / \cos \alpha)^2} \rightarrow \frac{\delta A}{\delta I} = \frac{\cos \alpha}{\cos \theta} \left(\frac{z}{f} \right)^2$$

Surface Radiance and Image Irradiance



Solid angle subtended by the lens,
as seen by the patch δA

$$\Omega = \frac{\pi}{4} \frac{d^2 \cos \alpha}{(z / \cos \alpha)^2} = \frac{\pi}{4} \left(\frac{d}{z} \right)^2 \cos^3 \alpha$$

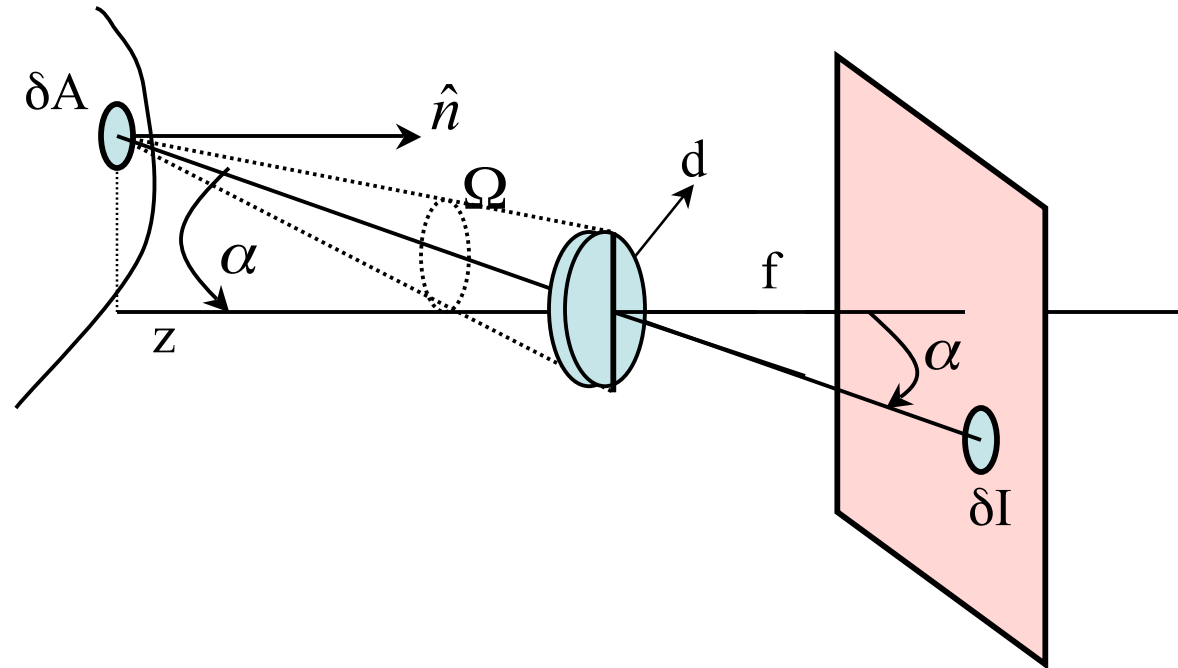
Power from patch δA through the lens

$$\delta P = L \Omega \delta A \cos \theta = L \delta A \frac{\pi}{4} \left(\frac{d}{z} \right)^2 \cos^3 \alpha \cos \theta$$

Thus, we conclude

$$E = \frac{\delta P}{\delta I} = L \frac{\delta A}{\delta I} \frac{\pi}{4} \left(\frac{d}{z} \right)^2 \cos^3 \alpha \cos \theta = L \frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha$$

Surface Radiance and Image Irradiance



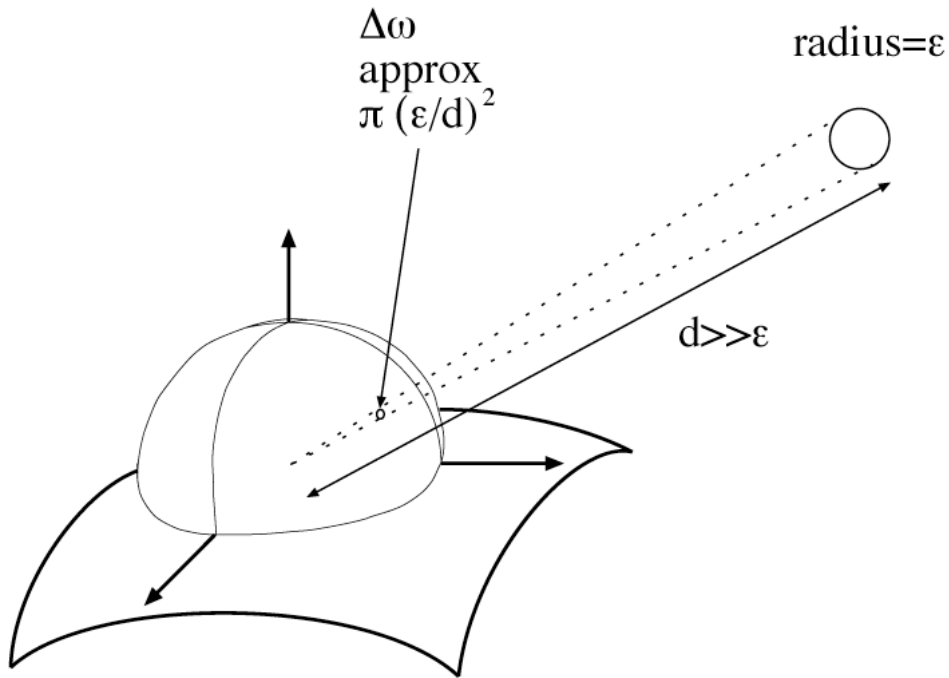
$$E = \frac{\delta P}{\delta I} = L \frac{\delta A}{\delta I} \frac{\pi}{4} \left(\frac{d}{z} \right)^2 \cos^3 \alpha \cos \theta = L \frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha$$

$$E \propto L_e$$

Image intensity is
proportional to Exident
Radiance

Radiance emitted by point sources

- small, distant sphere
radius ϵ and uniform
radiance E , which is far
away and subtends a
solid angle of about $\Omega = \pi \left(\frac{\epsilon}{d} \right)^2$



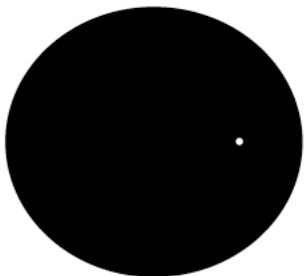
$$\Omega = \pi \left(\frac{\epsilon}{d} \right)^2$$

$$L_e = \rho_d(x) \int L_i(x, \omega) \cos \theta_i d\omega$$

$$= \rho_d(x) \int_{\Omega} L_i d\omega \cos \theta_i$$

$$\approx \rho_d(x) \left(\frac{\epsilon}{r(x)} \right)^2 E \cos \theta_i$$

$$= k \frac{\rho_d(x) \cos \theta_i}{r(x)^2}$$



Constant
radiance patch
due to source

Standard nearby point source model

- N is the surface normal
- rho is diffuse albedo
- S is source vector - a vector from x to the source, whose length is the intensity term
 - works because a dot-product is basically a cosine

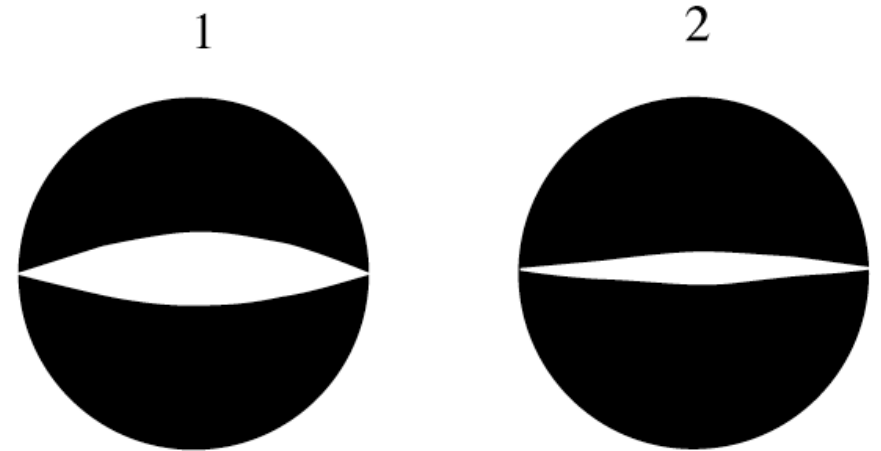
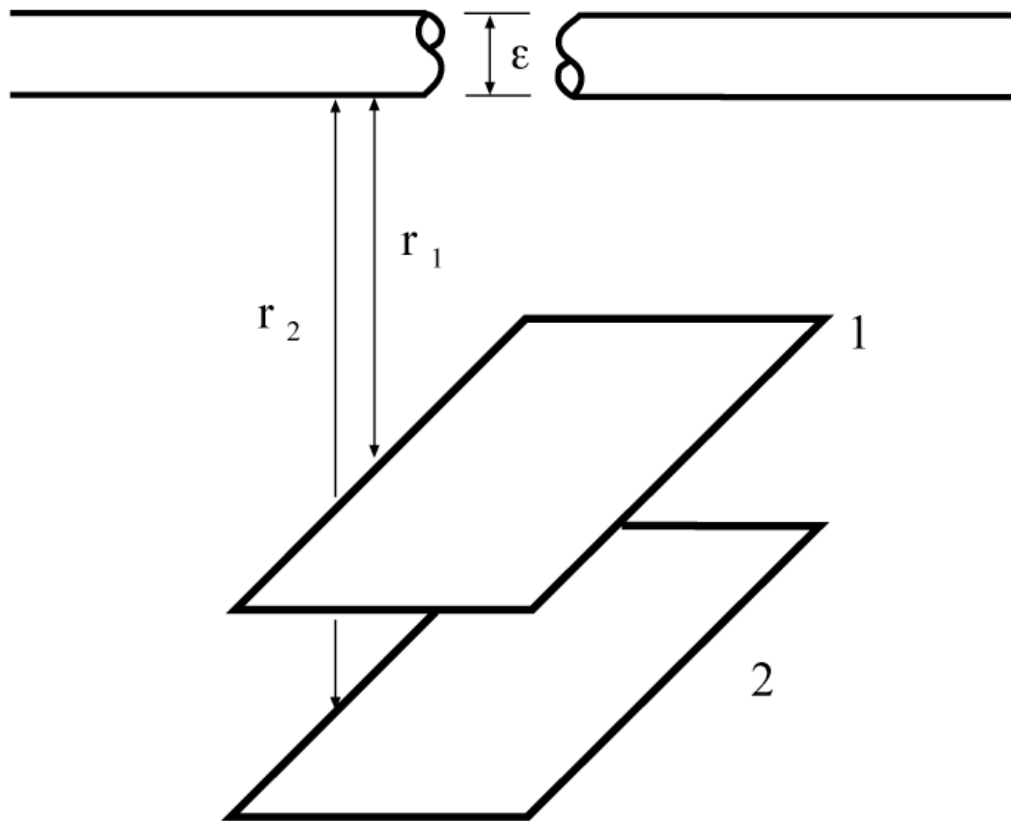
$$\rho_d(x) \left(\frac{N(x) \cdot S(x)}{r(x)^2} \right)$$

Standard distant point source model

- Issue: nearby point source gets bigger if one gets closer
 - the sun doesn't for any reasonable binding of closer
- Assume that all points in the model are close to each other with respect to the distance to the source. Then the source vector doesn't vary much, and the distance doesn't vary much either, and we can roll the constants together to get:

$$\rho_d(x)(N(x) \bullet S_d(x))$$

Line sources

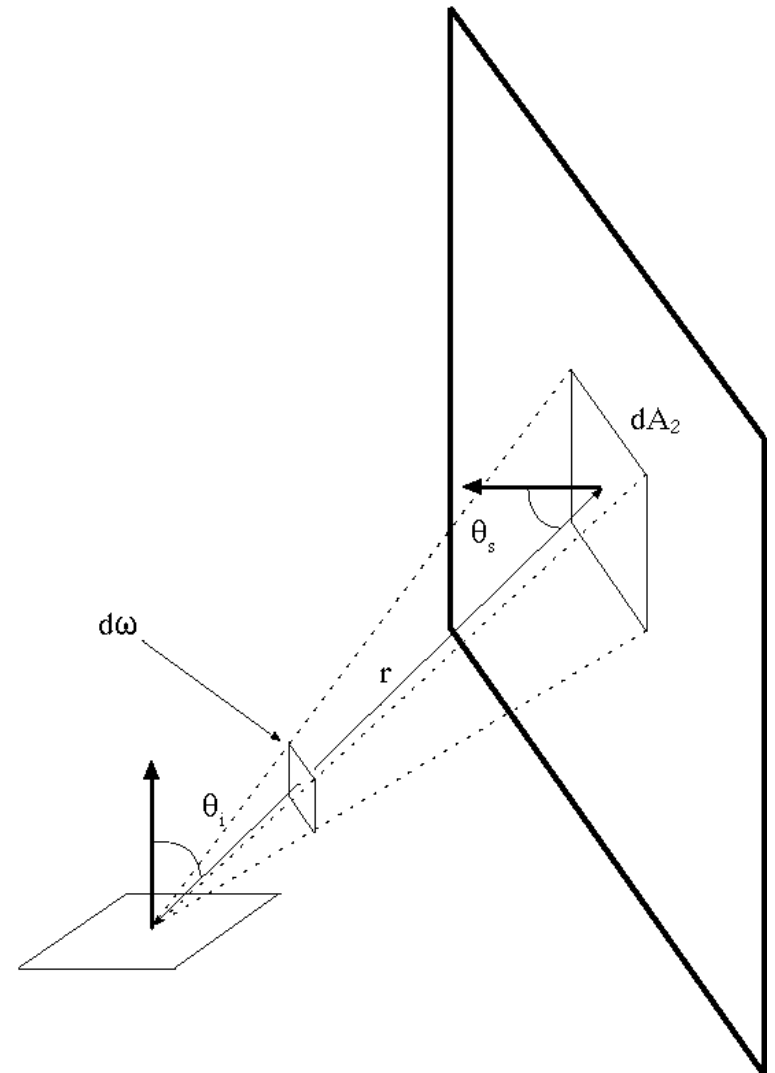


For a finite line source centered on the plane, there is spatial variation across the surface

radiosity due to line source varies with inverse distance, if the source is long enough.

Area sources

- Examples: diffuser boxes, white walls.
- The radiosity at a point due to an area source is obtained by adding up the contribution over the section of view hemisphere subtended by the source
 - change variables and add up over the source

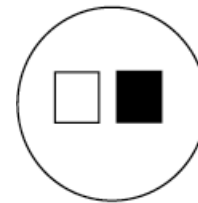


Area Source Shadows

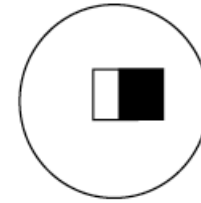
Area
Source



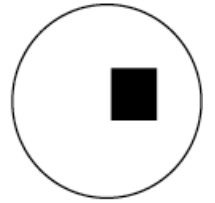
Occluder



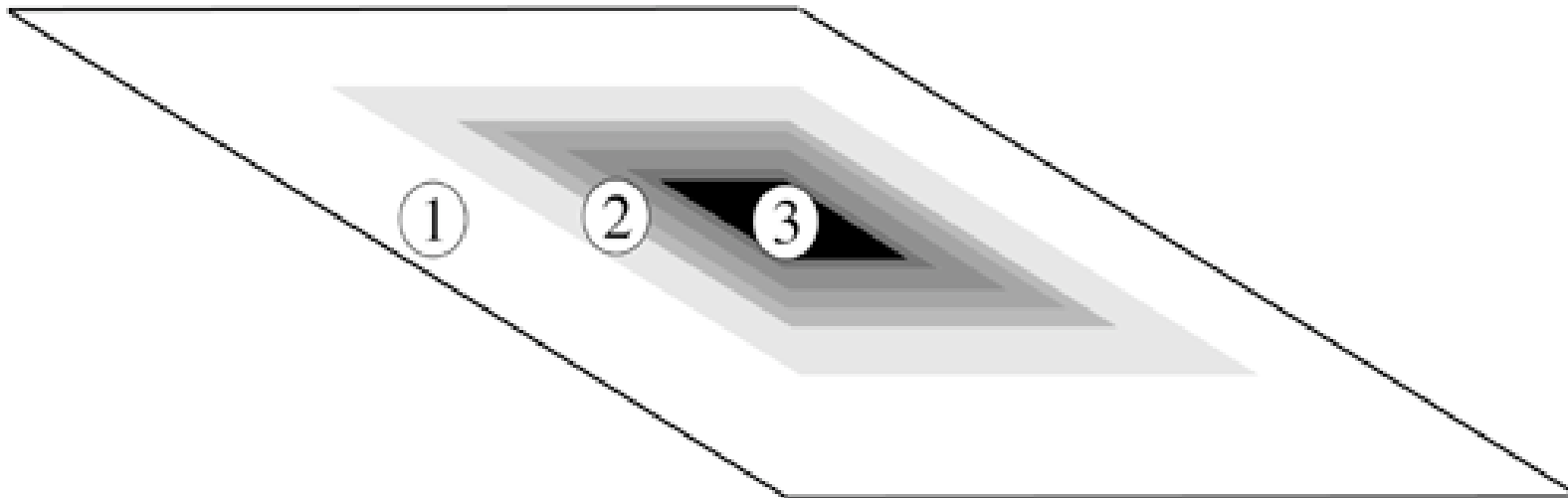
1



2



3



Shading models

- Local shading model
 - Surface has radiosity due only to sources visible at each point
 - Advantages:
 - often easy to manipulate, expressions easy
 - supports quite simple theories of how shape information can be extracted from shading
- Global shading model
 - surface radiosity is due to radiance reflected from other surfaces as well as from surfaces
 - Advantages:
 - usually very accurate
 - Disadvantage:
 - extremely difficult to infer anything from shading values

Photometric stereo

- Assume:
 - a local shading model
 - a set of point sources that are infinitely distant
 - a set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
 - A Lambertian object (or the specular component has been identified and removed)

Projection model for surface recovery - usually called a Monge patch

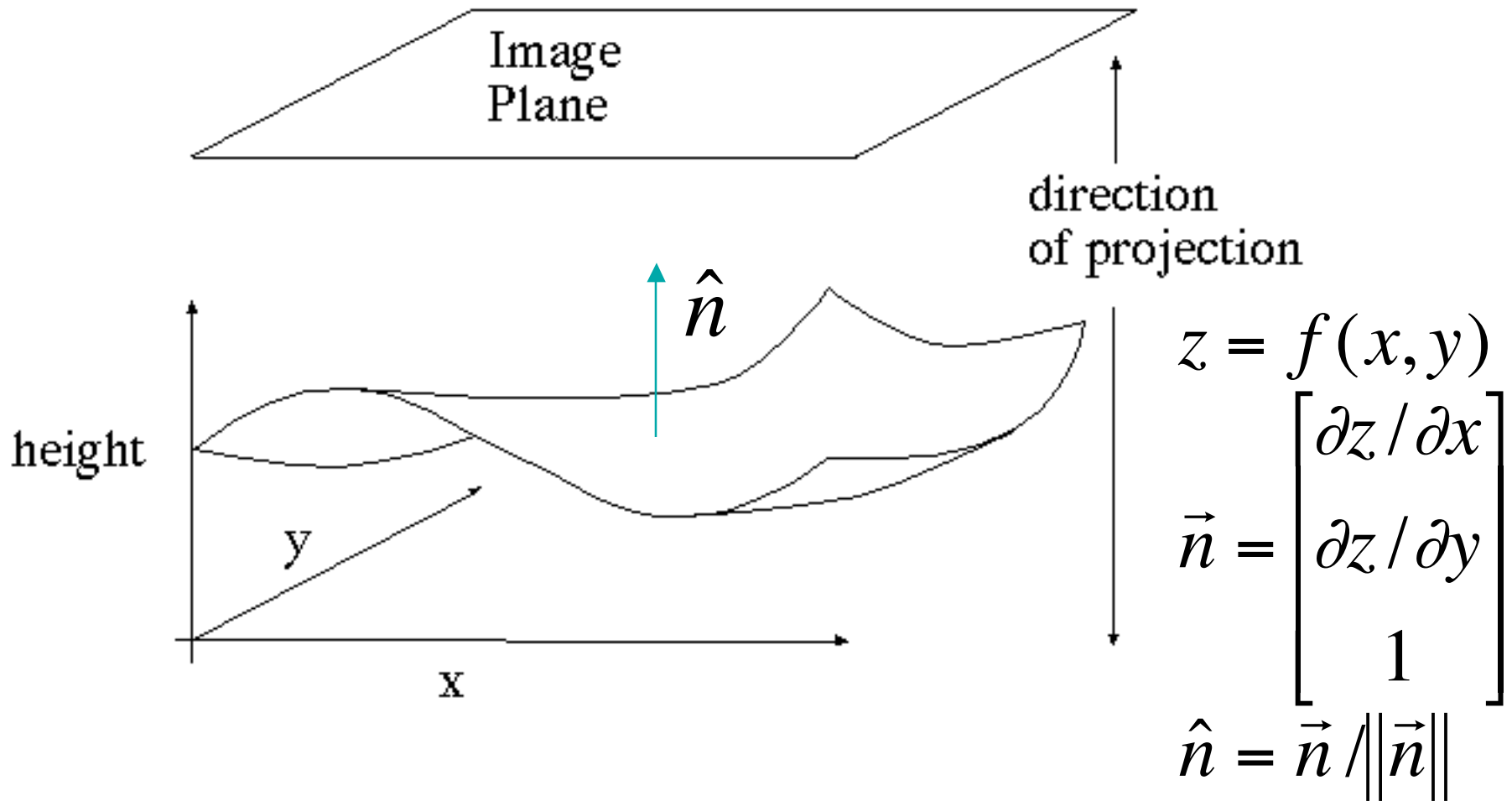


Image model

- For each point source, we know the source vector (by assumption). We assume we know the scaling constant of the linear camera. Fold the normal and the reflectance into one vector \mathbf{g} , and the scaling constant and source vector into another \mathbf{V}_j

- Out of shadow:

$$\begin{aligned} I_j(x, y) &= kB(x, y) \\ &= k\rho(x, y)(\mathbf{N}(x, y) \cdot \mathbf{S}_j) \\ &= \mathbf{g}(x, y) \cdot \mathbf{V}_j \end{aligned}$$

- In shadow:

$$I_j(x, y) = 0$$

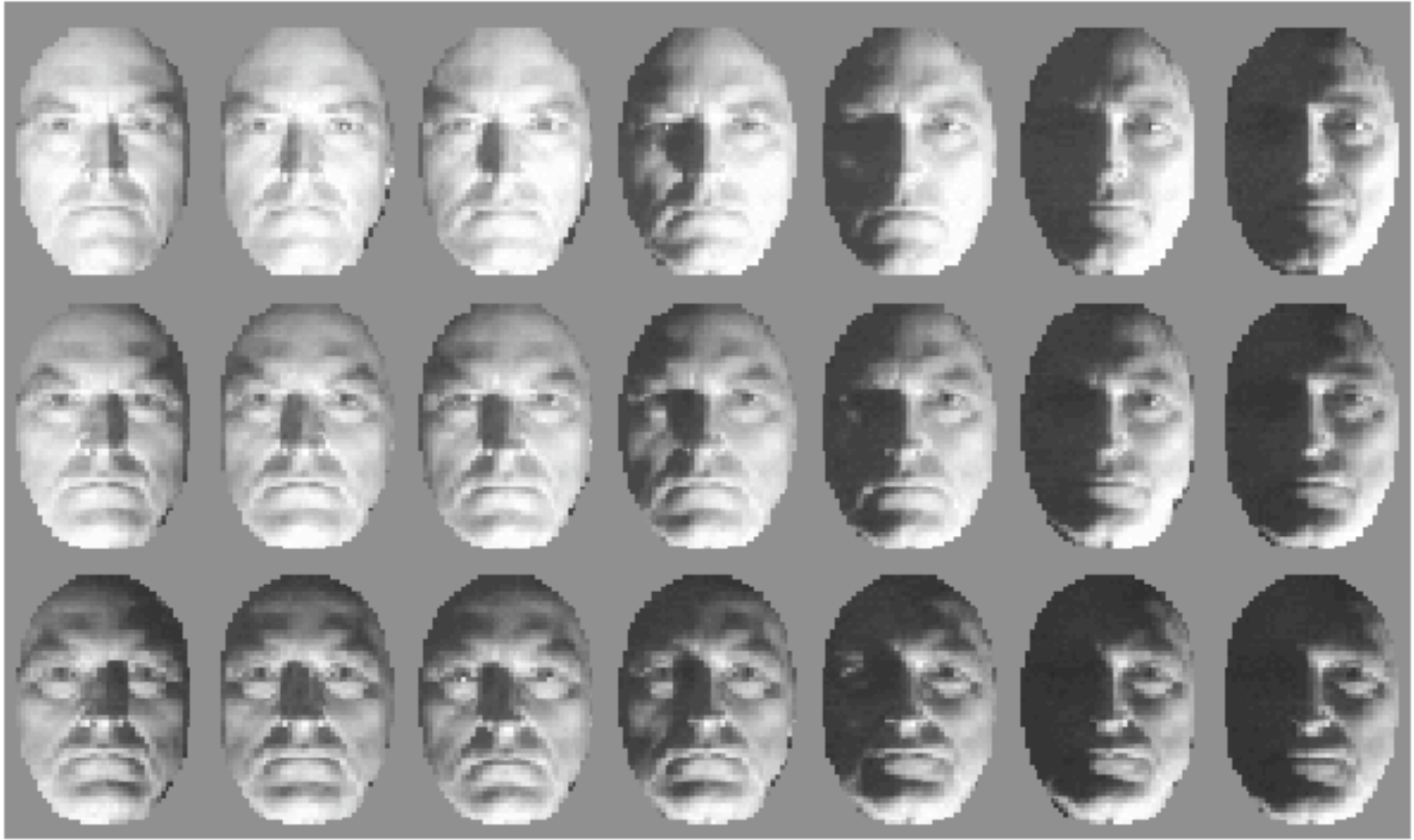
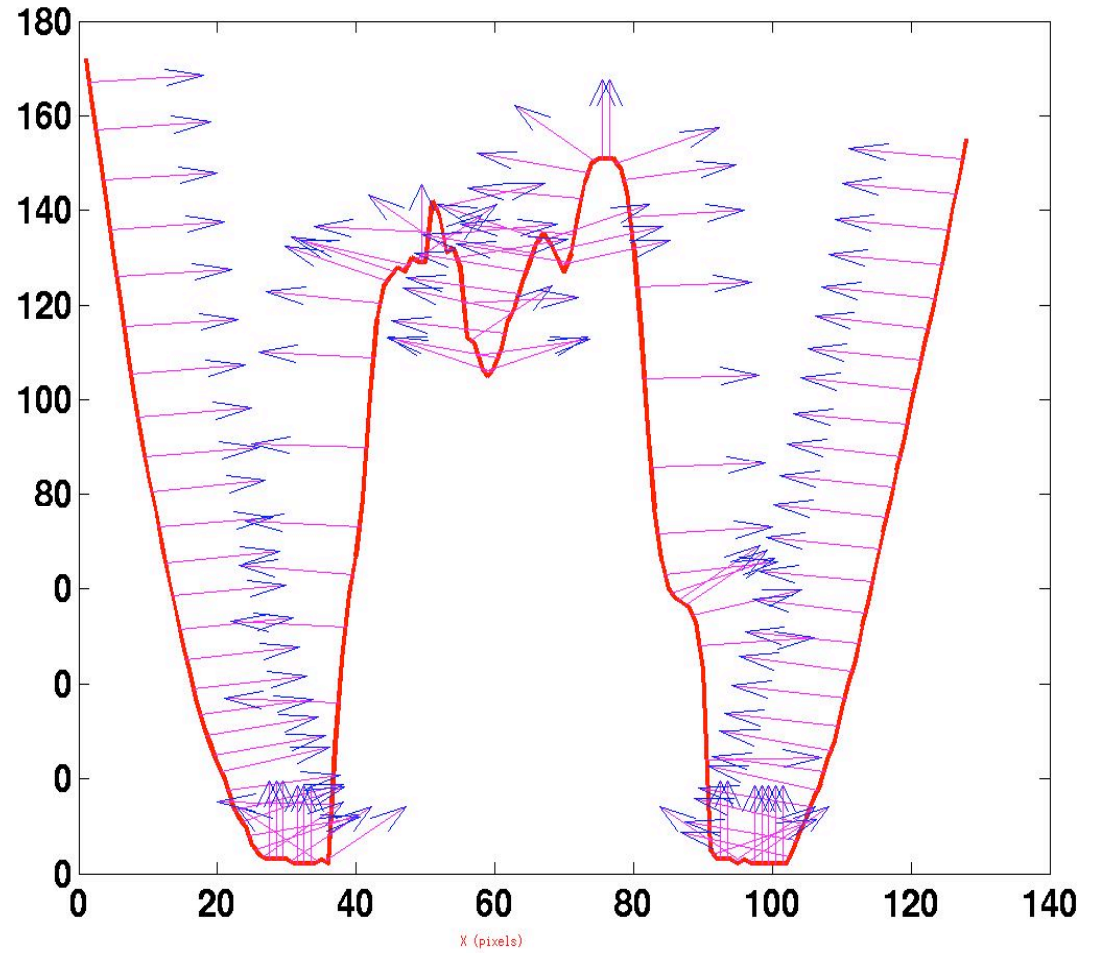
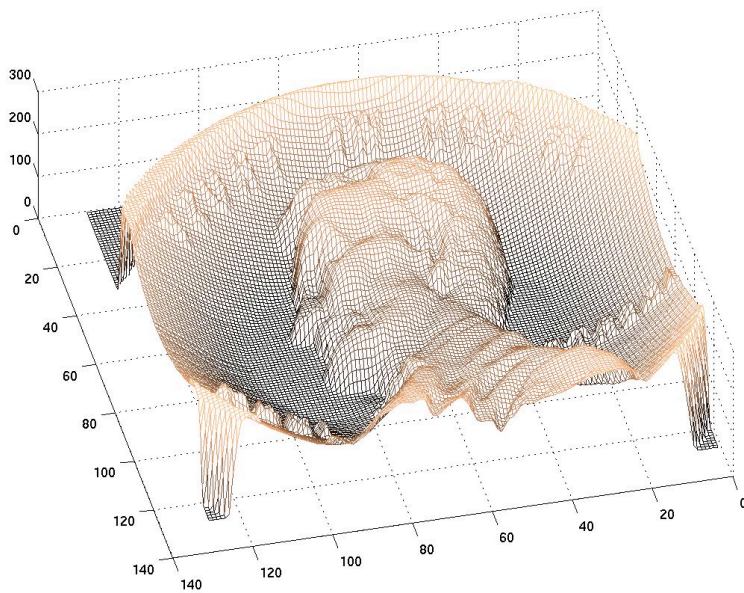
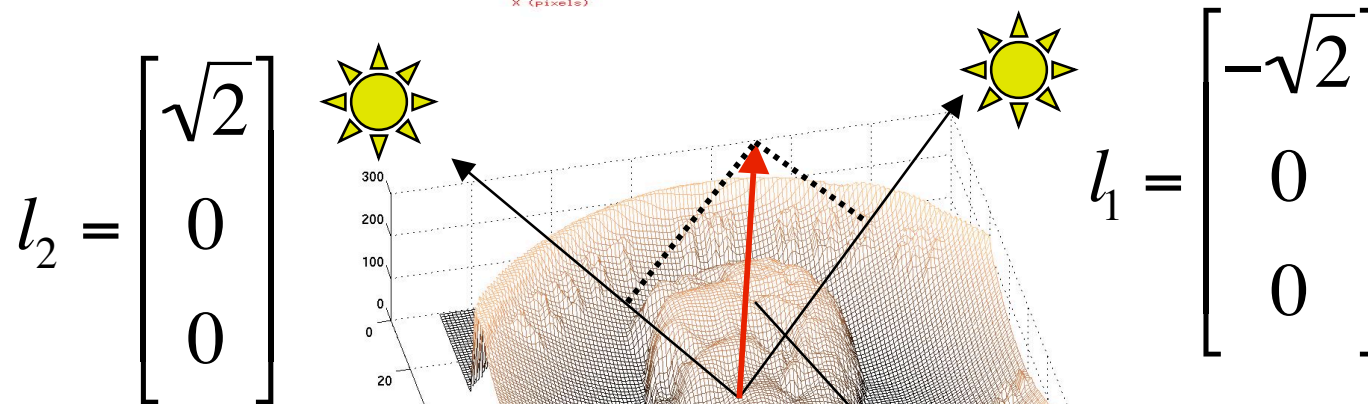
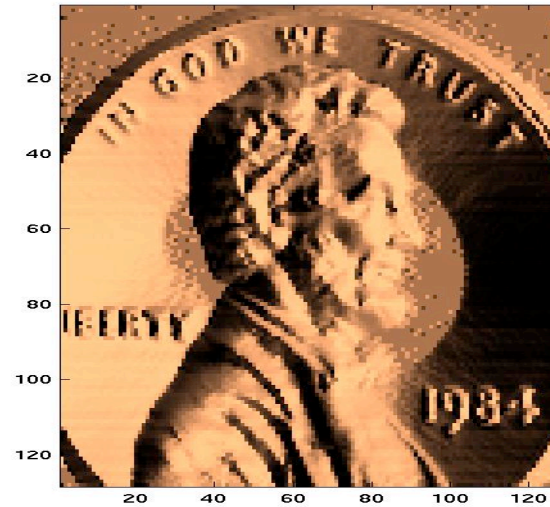
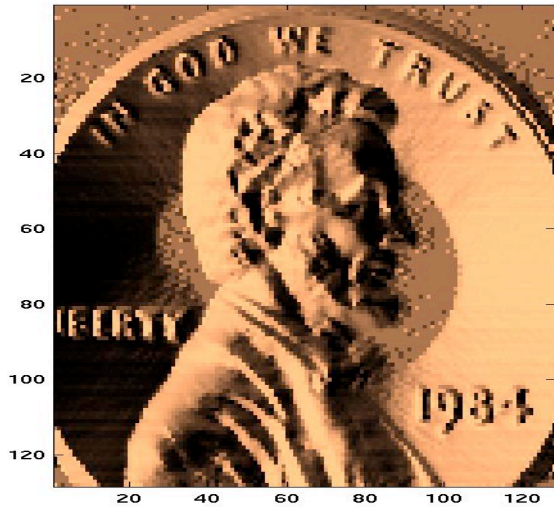


Figure 1: Examples of faces under different lighting conditions.

Surface normals





Light directions

Defined relative to local surface patch

Surface Normal

Basic Equation

$$\begin{matrix} n \times 1 \\ \left(\begin{array}{c} I_1(x,y) \\ I_2(x,y) \\ \dots \\ I_n(x,y) \end{array} \right) \end{matrix} = \max \left[\begin{matrix} n \times 3 \\ \left(\begin{array}{c} \mathbf{V}_1^T \\ \mathbf{V}_1^T \\ \dots \\ \mathbf{V}_n^T \end{array} \right) \mathbf{g}(x,y), \vec{0} \end{matrix} \right]$$

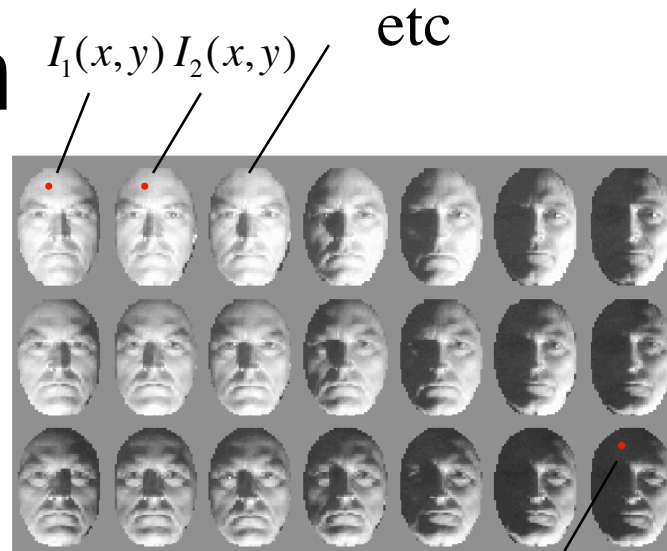


Figure 1: Examples of faces under different lighting conditions.

Values
Of images at
Pixel (x,y)

Matrix stack
Of light sources
For each of the n
images

Unknown

Problem- Nonlinear due to shadowing

Dealing with shadows

$$\begin{array}{ccccccc}
 n \times 1 & & & & & & n \times 3 \\
 \left(\begin{array}{c} I_1^2(x, y) \\ I_2^2(x, y) \\ \dots \\ I_n^2(x, y) \end{array} \right) & = & \left(\begin{array}{ccccc} I_1(x, y) & 0 & \dots & 0 \\ 0 & I_2(x, y) & \dots & \dots \\ \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & I_n(x, y) \end{array} \right) & \left(\begin{array}{c} \mathbf{V}_1^T \\ \mathbf{V}_2^T \\ \dots \\ \mathbf{V}_n^T \end{array} \right) & \mathbf{g}(x, y) \\
 | & & | & & / & & | \\
 \text{Known} & & \text{Known} & & \text{Known} & & \text{Unknown}
 \end{array}$$

General form: $\vec{b} = A\vec{x}$ *For each x,y point*

Recovering normal and reflectance

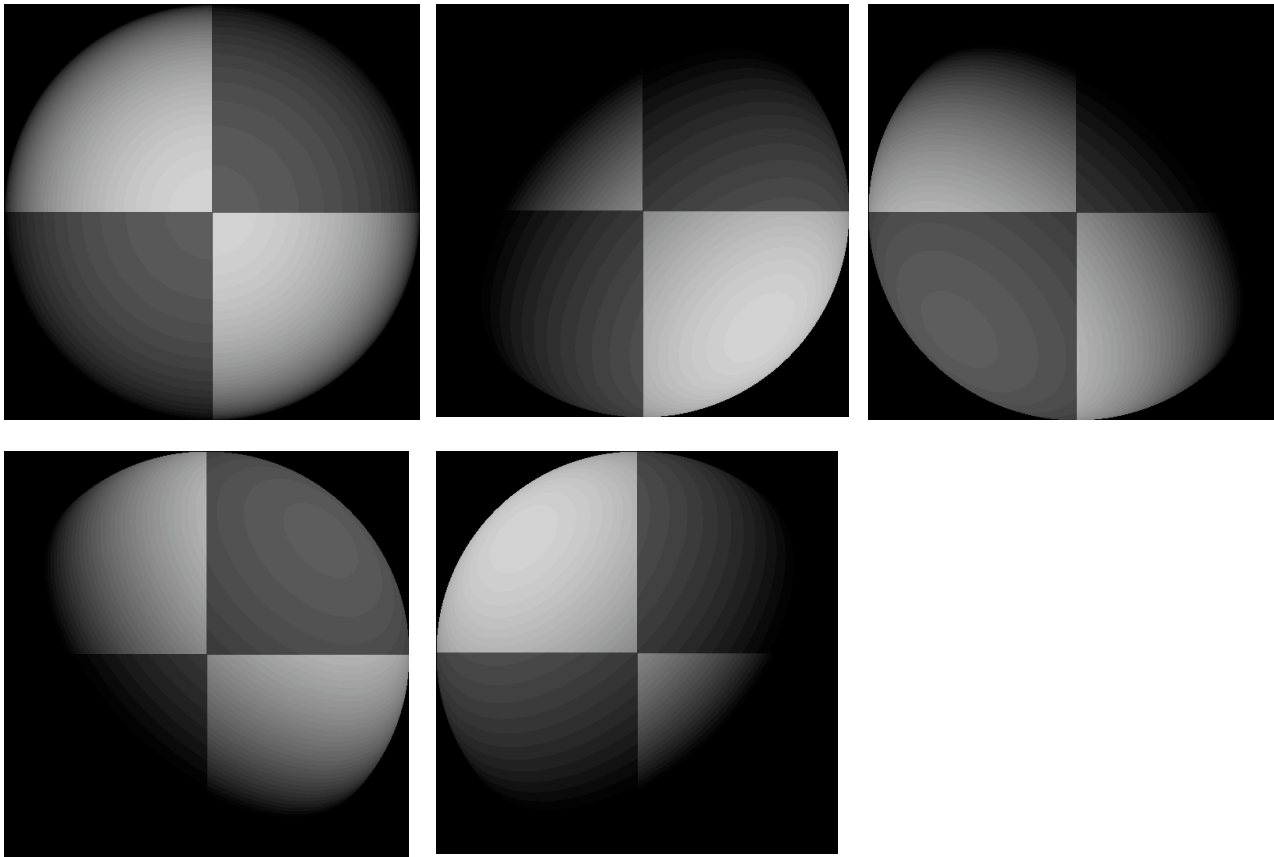
- Given sufficient sources, we can solve the previous equation (most likely need a least squares solution) for

$$\mathbf{g}(x, y) = \rho(x,y) \mathbf{N}(x, y)$$

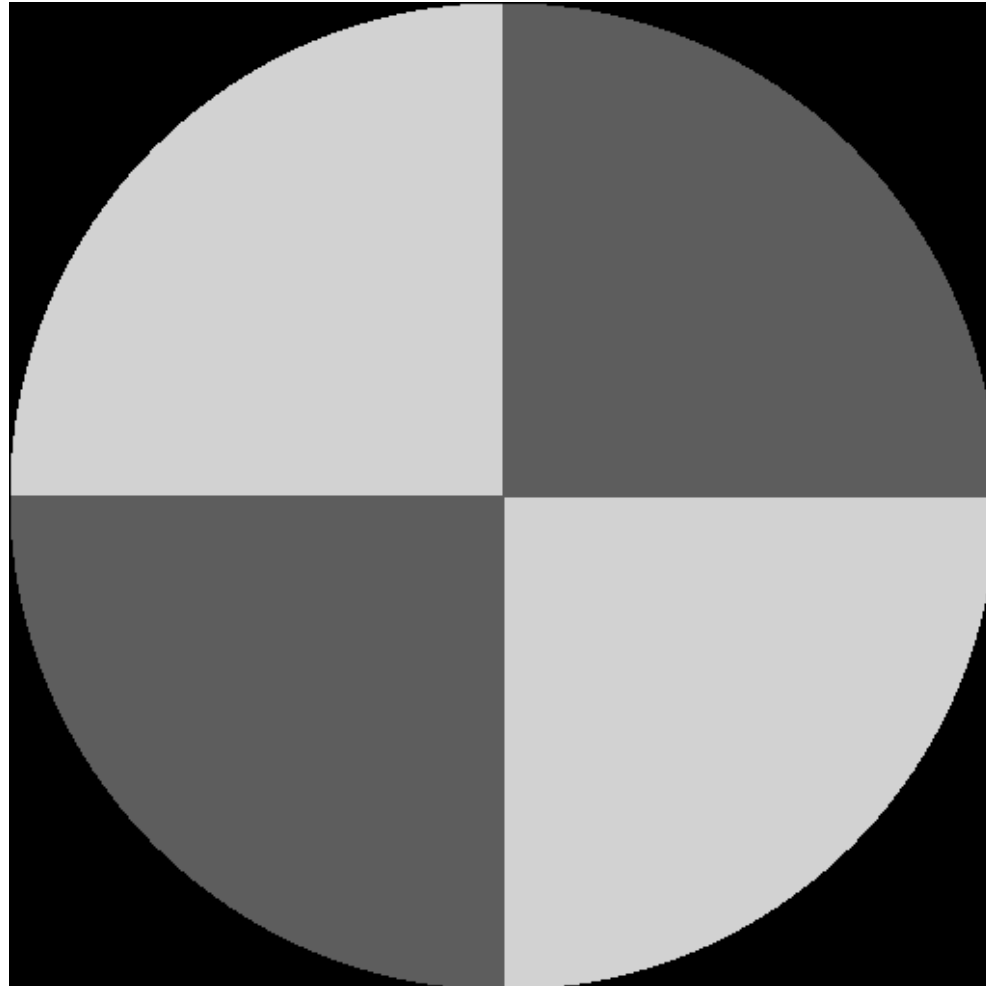
- Recall that $\mathbf{N}(x, y)$ is the unit normal
- This means that $\rho(x,y)$ is the magnitude of $\mathbf{g}(x, y)$
- This yields a check
 - If the magnitude of $\mathbf{g}(x, y)$ is greater than 1, there's a problem
- And

$$\mathbf{N}(x, y) = \mathbf{g}(x, y) / \rho(x,y)$$

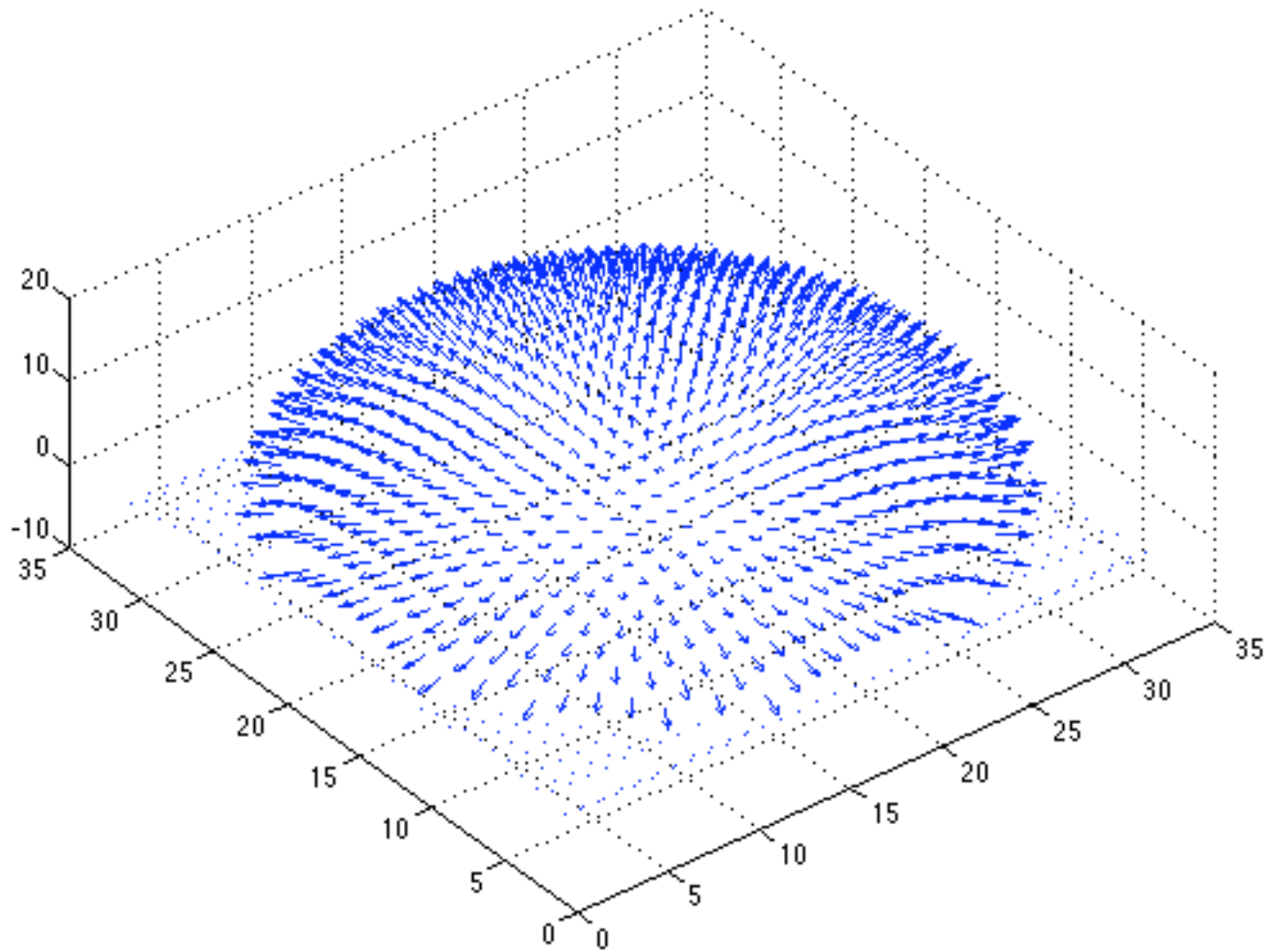
Example figures



Recovered reflectance



Recovered normal field



Recovering a surface from normals - 1

- Recall the surface is written as

$$(x, y, f(x, y))$$

- This means the normal has the form:

$$N(x, y) = \left(\frac{1}{\sqrt{f_x^2 + f_y^2 + 1}} \right) \begin{pmatrix} -f_x \\ -f_y \\ 1 \end{pmatrix}$$

- If we write the known vector \mathbf{g} as

$$\mathbf{g}(x, y) = \begin{pmatrix} g_1(x, y) \\ g_2(x, y) \\ g_3(x, y) \end{pmatrix}$$

- Then we obtain values for the partial derivatives of the surface:

$$f_x(x, y) = (g_1(x, y) / g_3(x, y))$$

$$f_y(x, y) = (g_2(x, y) / g_3(x, y))$$

Recovering a surface from normals - 2

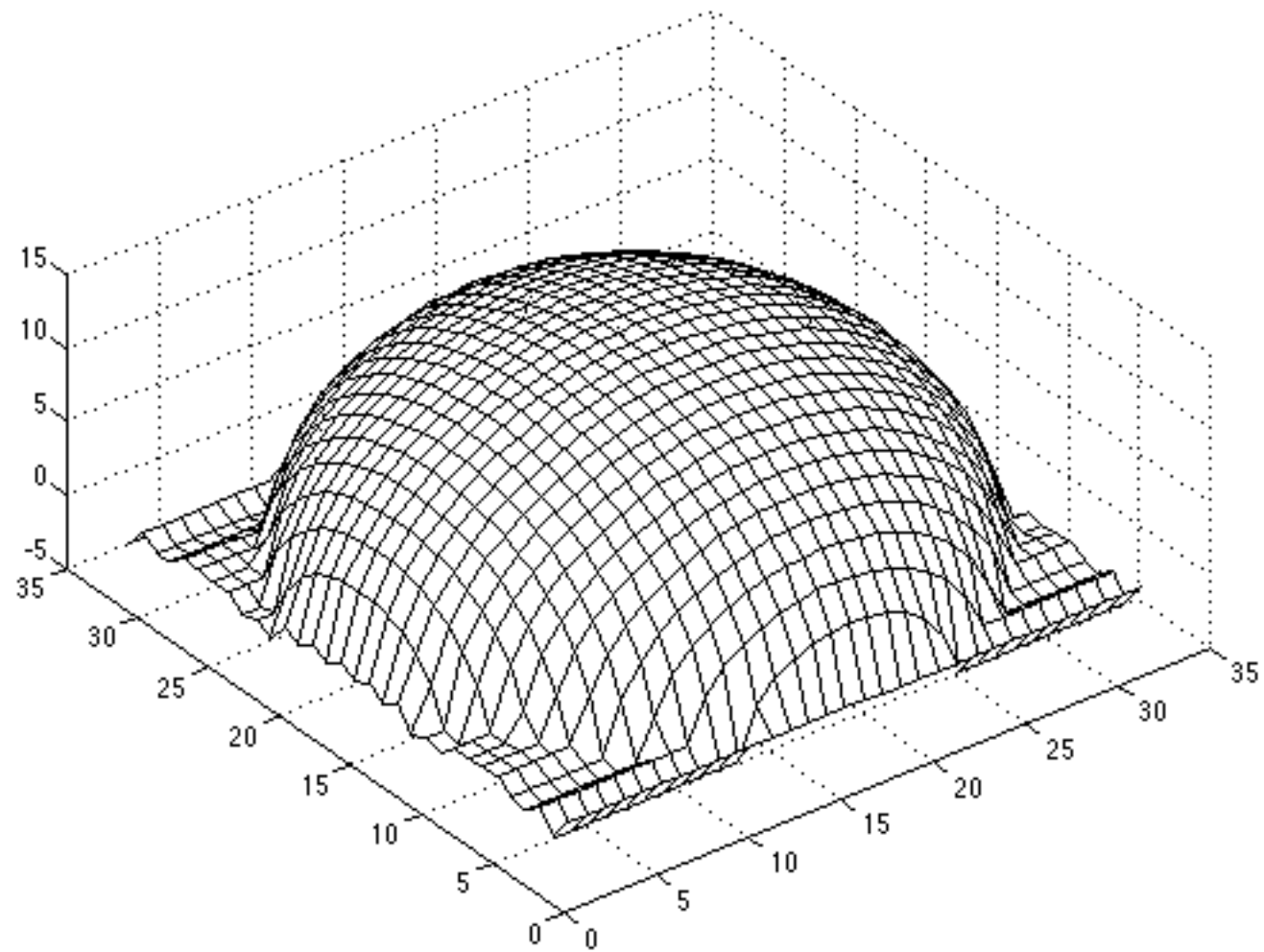
- Recall that mixed second partials are equal --- this gives us a **check**. We must have:
- We can now recover the surface height at any point by integration along some path, e.g.

$$\frac{\frac{\partial(g_1(x, y)/g_3(x, y))}{\partial y}}{\frac{\partial(g_2(x, y)/g_3(x, y))}{\partial x}} =$$

(or they should be similar, at least)

$$f(x, y) = \int_0^x f_x(s, y) ds + \int_0^y f_y(x, t) dt + c$$

Surface recovered by integration



Shape from Shading with ambiguity

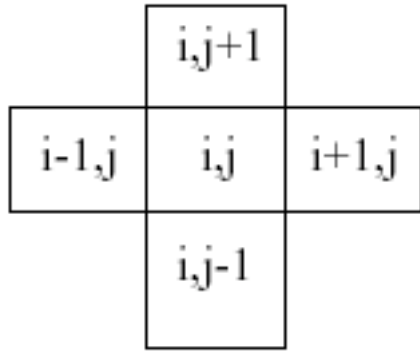
What to do if light source unknown?

Only one image?

Ans: Use prior knowledge in the form of constraints (e.g. regularization methods, Bayesian methods), take advantage of the *habits* (regularities) of images.

Horn's approach

- Want to estimate scene parameters (surface slopes $f_x(x,y)$ and $f_y(x,y)$ at every image position, (x,y)).
- Have a rendering function that takes you from some given set of scene parameters to observation data (e.g. $r(x,y)$ $n(x,y)$ gives image intensity for any (x,y)).
- Could try to find the parameters $f_x(x,y)$ & $f_y(x,y)$ that minimize the difference from the observations $I(x,y)$.
- But the problem is “ill-posed”, or underspecified from that constraint alone. So add-in additional requirements that the scene parameters must satisfy (the surface slopes $f_x(x,y)$ & $f_y(x,y)$ must be smooth at every point).



Regularization

For each normal, compute the distance from the normal to its neighbors:

Prior/regularizer

$$s(i, j) = \sum_{l=\{-1,1\}} \sum_{k=\{-1,1\}} \left(\vec{n}(i+k, j+l) - \vec{n}(i, j) \right)^2$$

Intensity Error

$$r(i, j) = \left(I(i, j) - I_{pred}(i, j) \right)^2$$

$$Err = \sum_{i,j} r(i, j) + \lambda s(i, j)$$

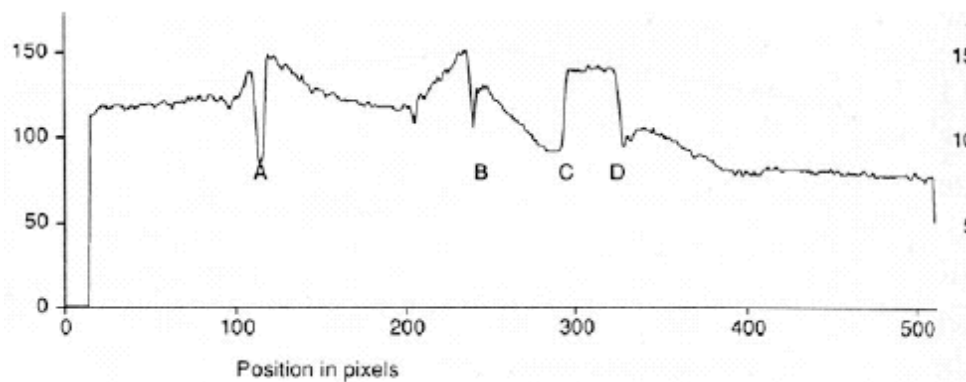
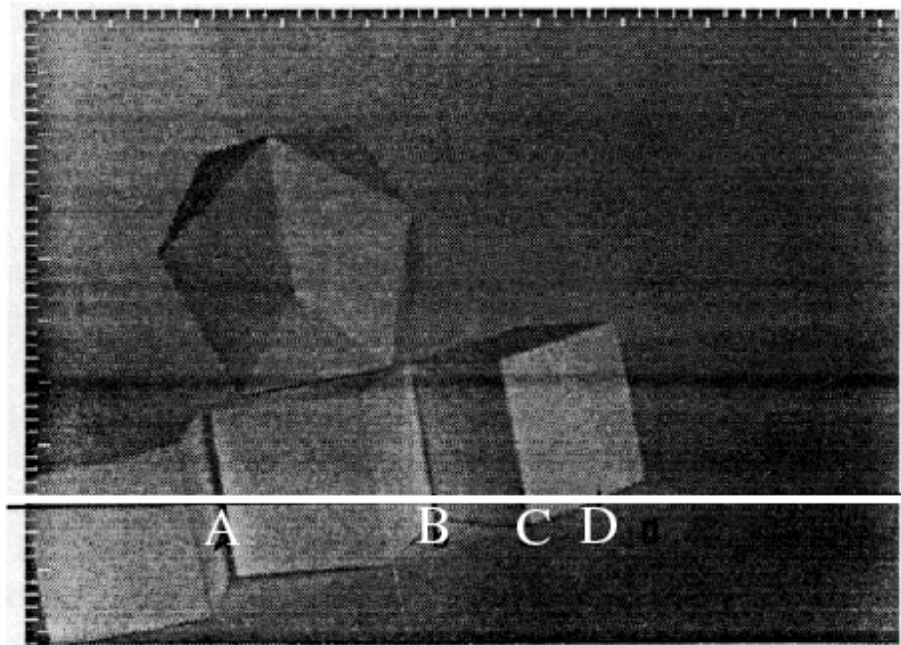
Shape from Shading Ambiguity



Curious Experimental Fact

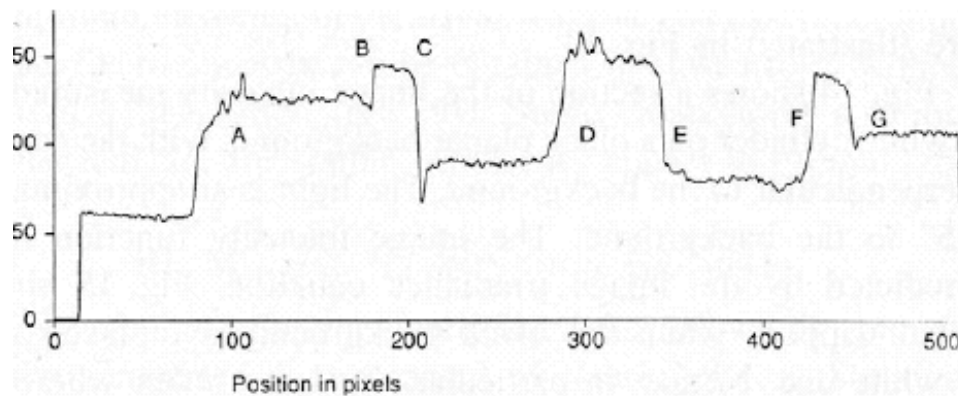
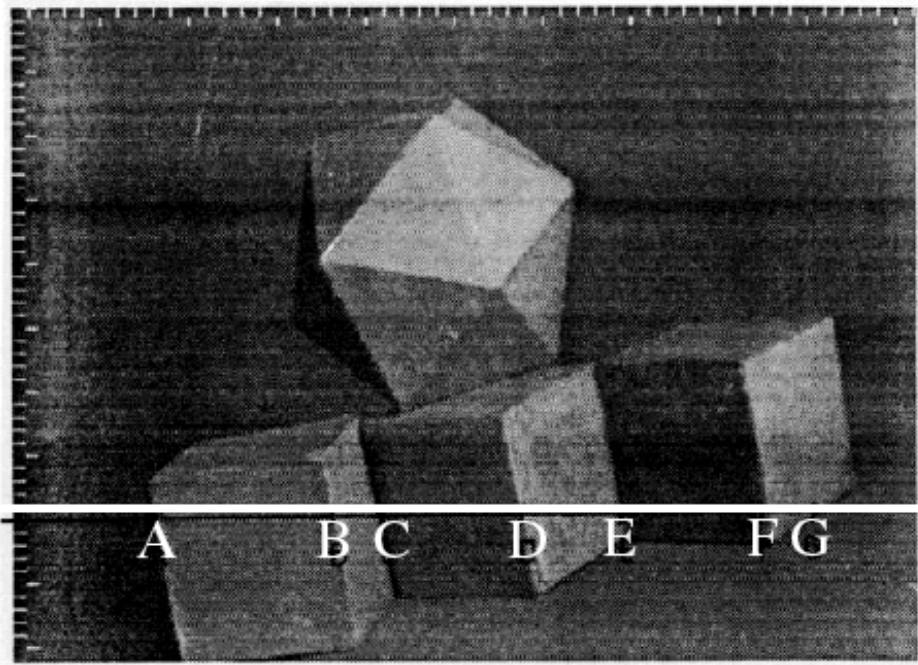
- Prepare two rooms, one with white walls and white objects, one with black walls and black objects
- Illuminate the black room with bright light, the white room with dim light
- People can tell which is which (due to Gilchrist)

- Why? (a local shading model predicts they can't).



A view of a white room, under dim light. Below, we see a cross-section of the image intensity corresponding to the line drawn on the image.

Figure from "Mutual Illumination," by D.A. Forsyth and A.P. Zisserman, Proc. CVPR, 1989, copyright 1989 IEEE



A view of a black room, under bright light. Below, we see a cross-section of the image intensity corresponding to the line drawn on the image.

What's going on here?

- local shading model is a poor description of physical processes that give rise to images
 - because surfaces reflect light onto one another
- This is a major nuisance; the distribution of light (in principle) depends on the configuration of every radiator; big distant ones are as important as small nearby ones (solid angle)
- The effects are easy to model
- It appears to be hard to extract information from these models

Interreflections - a global shading model

- Other surfaces are now area sources - this yields:

Radiosity at surface = Exitance + Radiosity due to other surfaces

$$B(x) = E(x) + \rho_d(x) \int_{\text{all other surfaces}} B(u) \frac{\cos \theta_i \cos \theta_s}{\pi r(x, u)^2} \text{Vis}(x, u) dA_u$$

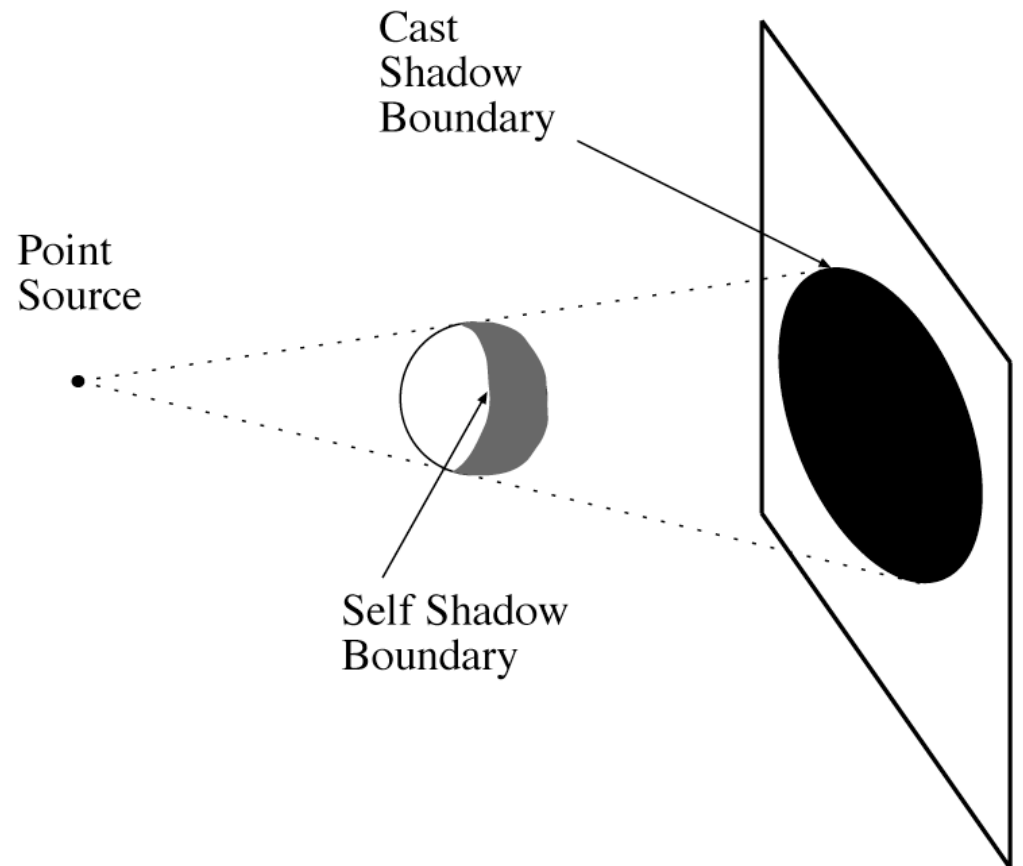
- $\text{Vis}(x, u)$ is 1 if they can see each other, 0 if they can't

What do we do about this?

- Attempt to build approximations
 - Ambient illumination
- Study qualitative effects
 - reflexes
 - decreased dynamic range
 - smoothing
- Try to use other information to control errors

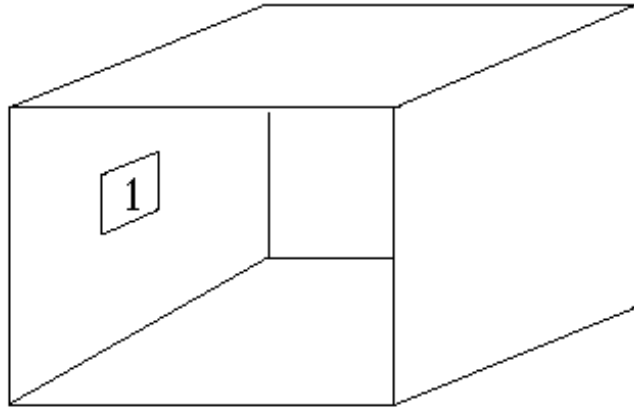
Shadows cast by a point source

- A point that can't see the source is in shadow
- For point sources, the geometry is simple

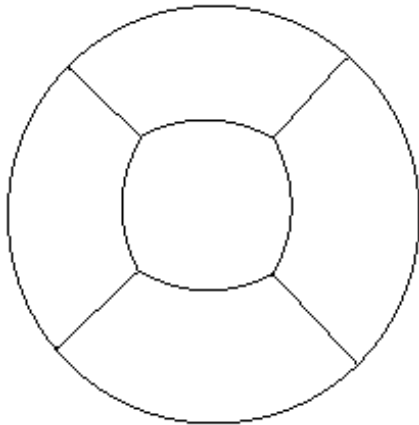
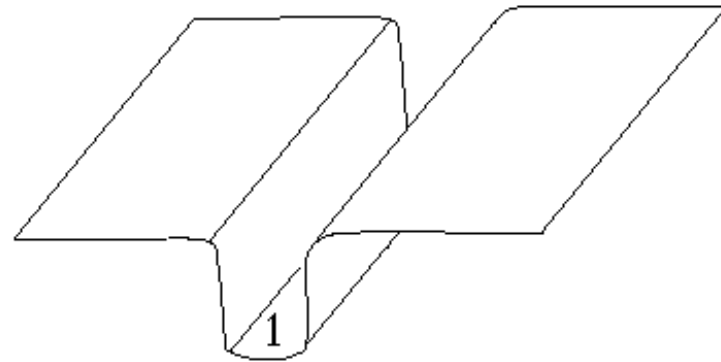


Ambient Illumination

- Two forms
 - Add a constant to the radiosity at every point in the scene to account for brighter shadows than predicted by point source model
 - Advantages: simple, easily managed (e.g. how would you change photometric stereo?)
 - Disadvantages: poor approximation (compare black and white rooms)
 - Add a term at each point that depends on the size of the clear viewing hemisphere at each point (see next slide)
 - Advantages: appears to be quite a good approximation, but jury is out
 - Disadvantages: difficult to work with



Infinite Plane



View from 1

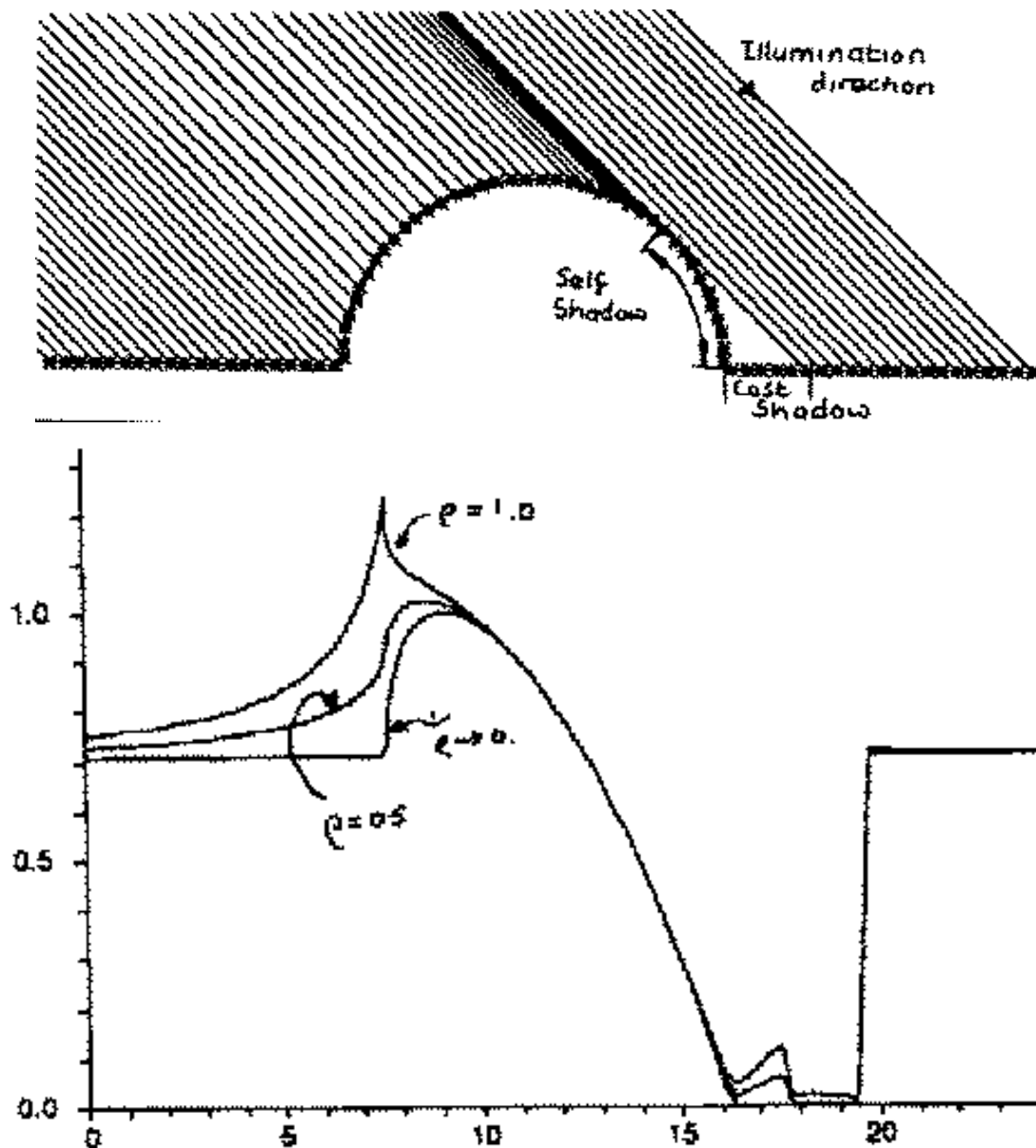


View from 1

At a point inside a cube or room, the surface sees light in all directions, so add a large term. At a point on the base of a groove, the surface sees relatively little light, so add a smaller term.

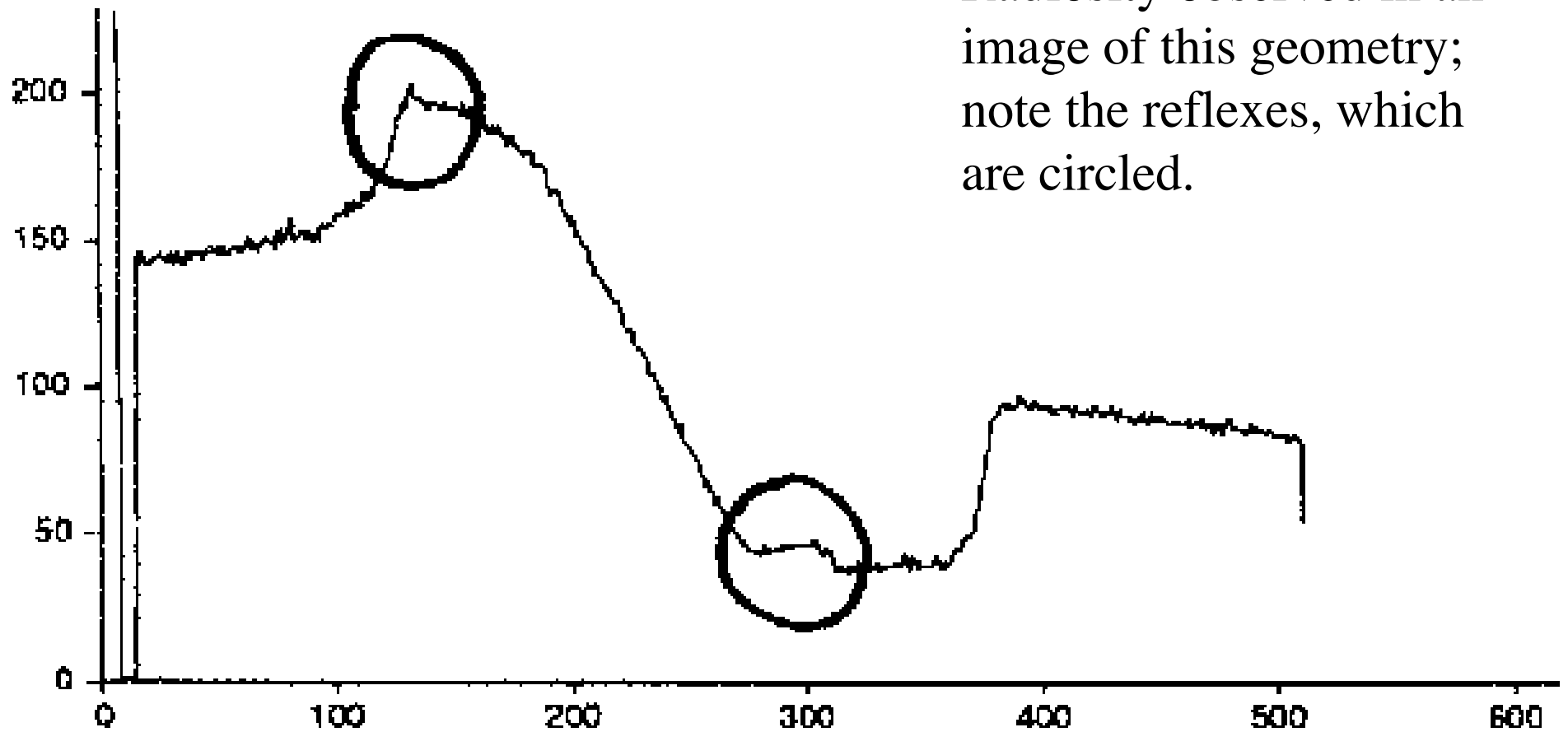
Reflexes

- A characteristic feature of interreflections is little bright patches in concave regions
 - Examples in following slides
 - Perhaps one should detect and reason about reflexes?
 - Known that artists reproduce reflexes, but often too big and in the wrong place



At the top, geometry of a semi-circular bump on a plane; below, predicted radiosity solutions, scaled to lie on top of each other, for different albedos of the geometry. When albedo is close to zero, shading follows a local model; when it is close to one, there are substantial reflexes.

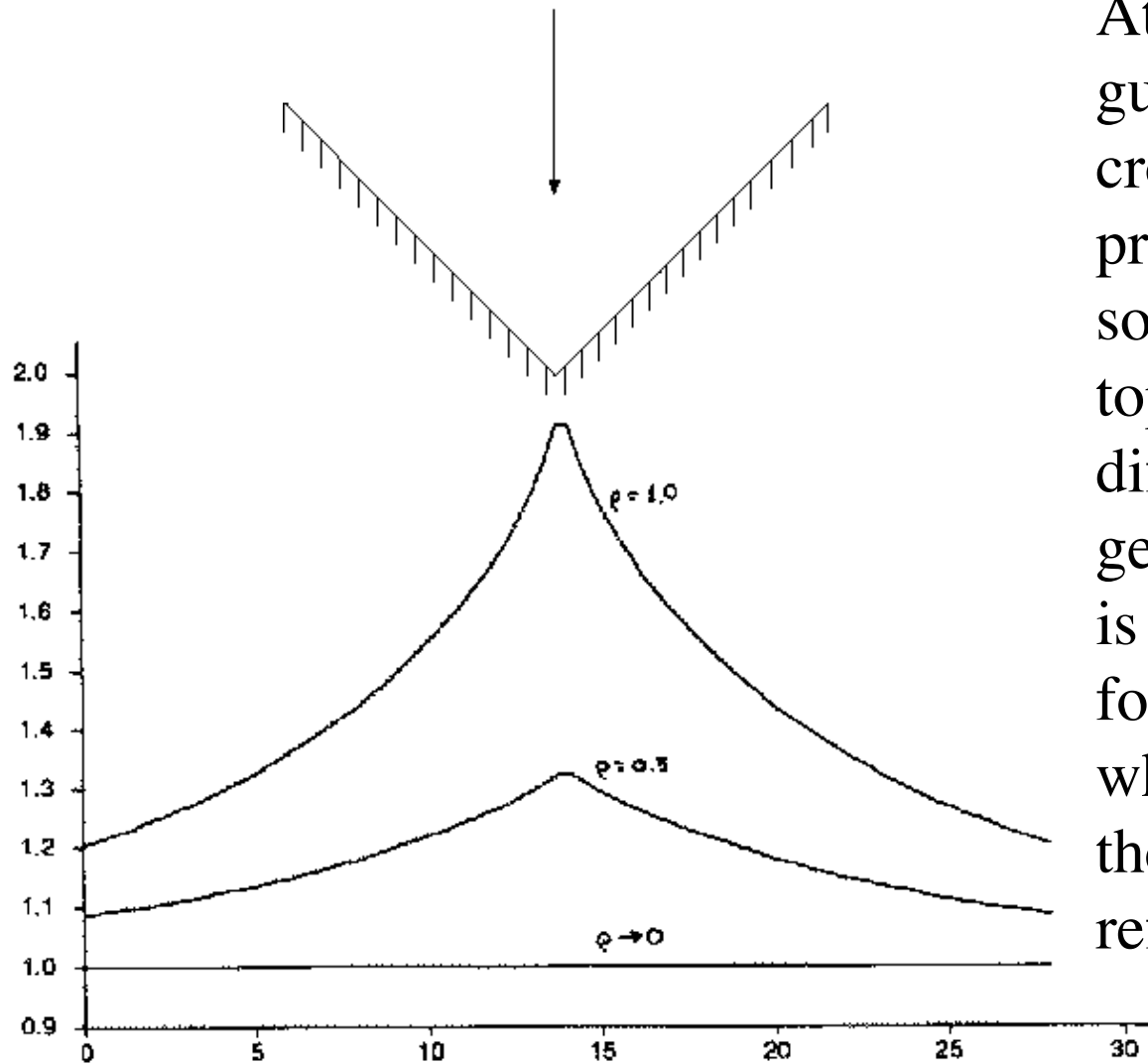
Figure from "Mutual Illumination," by D.A. Forsyth and A.P. Zisserman, Proc. CVPR, 1989, copyright 1989 IEEE



Radiosity observed in an image of this geometry; note the reflexes, which are circled.

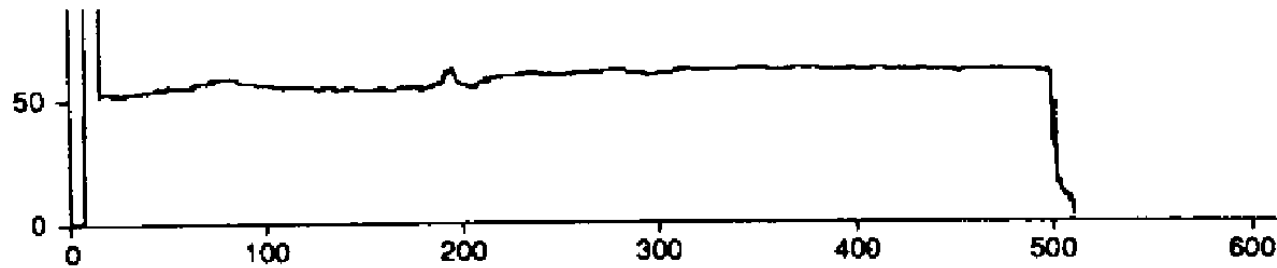
Figure from "Mutual Illumination," by D.A. Forsyth and A.P. Zisserman, Proc. CVPR, 1989, copyright 1989 IEEE

Illumination from
an infinitely distant
point source, in this
direction



At the top, geometry of a gutter with triangular cross-section; below, predicted radiosity solutions, scaled to lie on top of each other, for different albedos of the geometry. When albedo is close to zero, shading follows a local model; when it is close to one, there are substantial reflexes.

Figure from "Mutual Illumination," by D.A. Forsyth and A.P. Zisserman, Proc. CVPR, 1989, copyright 1989 IEEE



Radiosity observed in an image of this geometry; above, for a black gutter and below for a white one

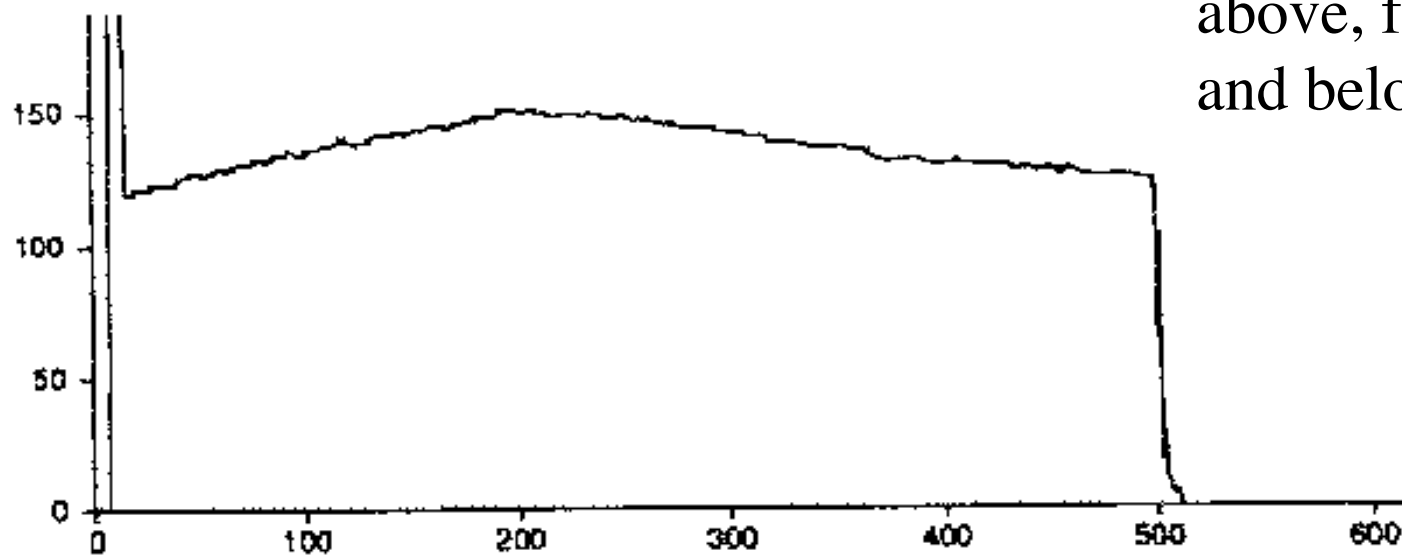
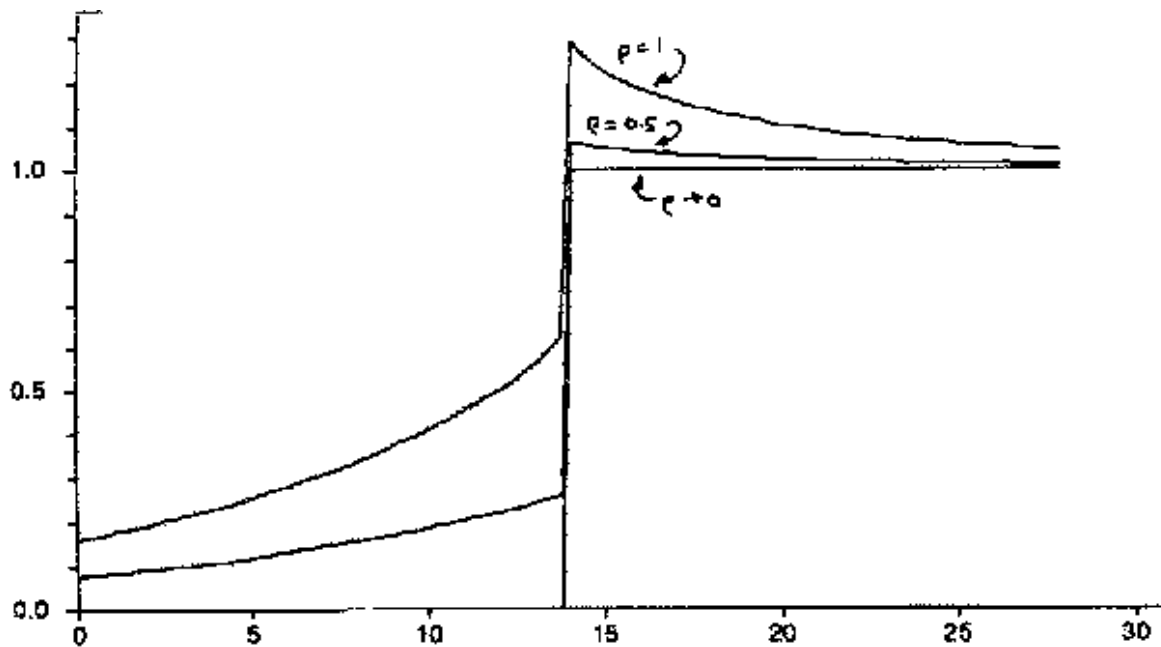
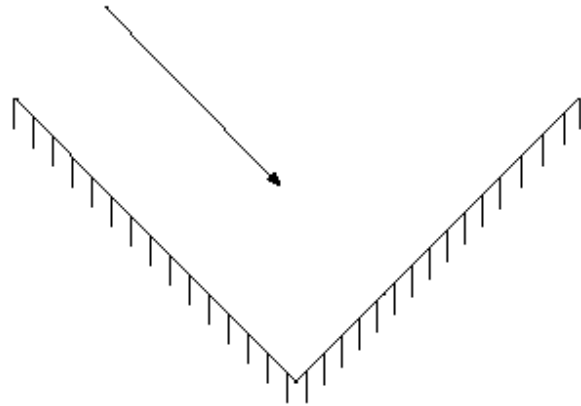


Figure from "Mutual Illumination," by D.A. Forsyth and A.P. Zisserman, Proc. CVPR, 1989, copyright 1989 IEEE

Illumination from
an infinitely distant
point source, in this
direction



At the top, geometry of a gutter with triangular cross-section; below, predicted radiosity solutions, scaled to lie on top of each other, for different albedos of the geometry. When albedo is close to zero, shading follows a local model; when it is close to one, there are substantial reflexes.

Figure from "Mutual Illumination," by D.A. Forsyth and A.P. Zisserman, Proc. CVPR, 1989, copyright 1989 IEEE

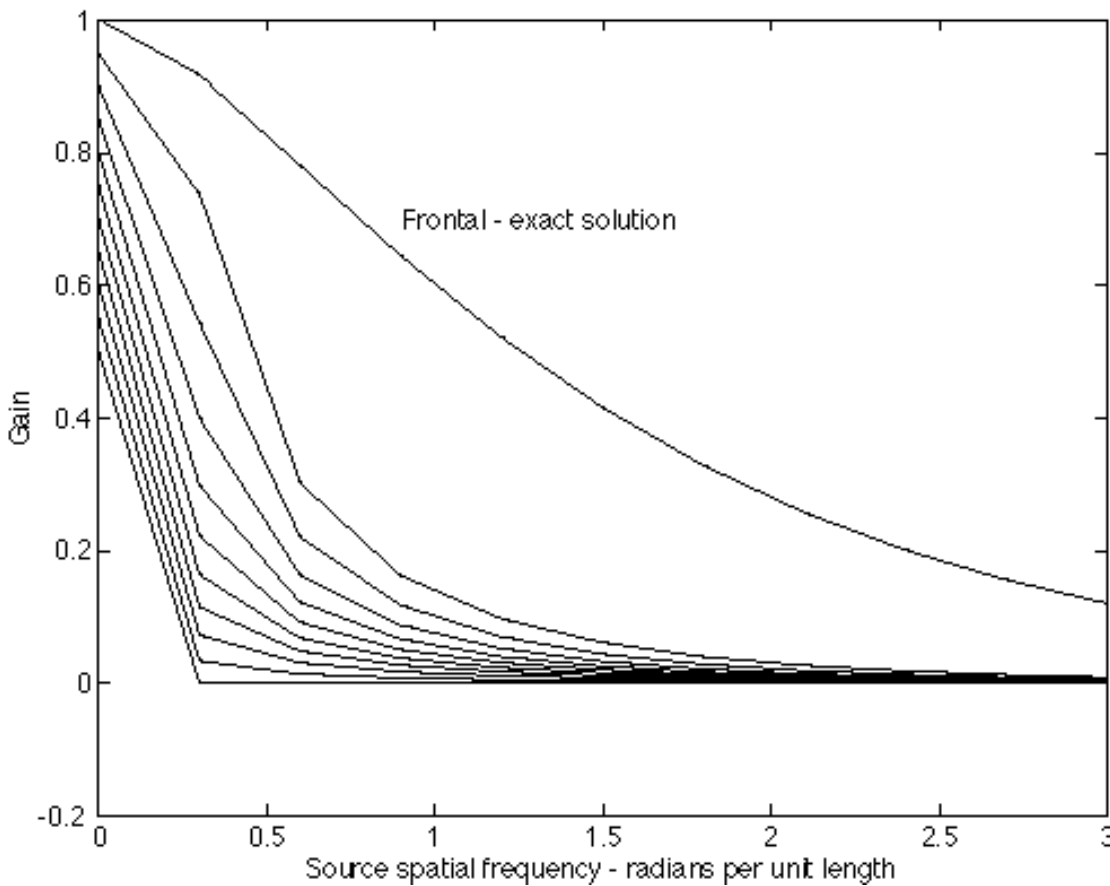
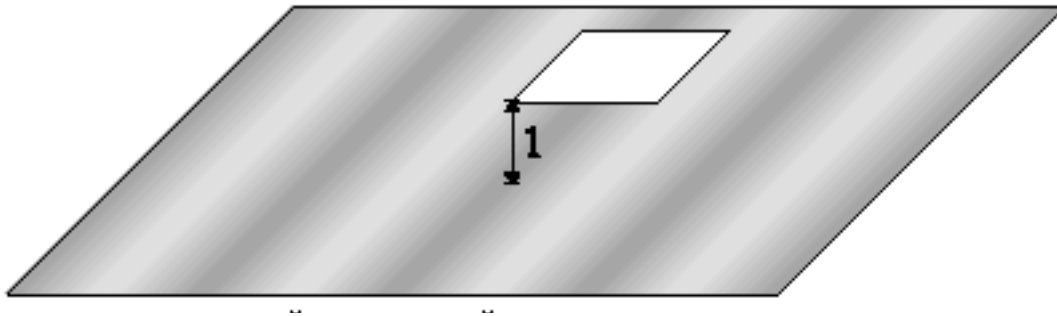


Radiosity observed in an image of this geometry for a white gutter.

Figure from "Mutual Illumination," by D.A. Forsyth and A.P. Zisserman, Proc. CVPR, 1989, copyright 1989 IEEE

Smoothing

- Interreflections smooth detail
 - E.g. you can't see the pattern of a stained glass window by looking at the floor at the base of the window; at best, you'll see coloured blobs.
 - This is because, as I move from point to point on a surface, the pattern that I see in my incoming hemisphere doesn't change all that much
 - Implies that fast changes in the radiosity are local phenomena.



Fix a small patch near a large radiator carrying a periodic radiosity signal; the radiosity on the surface is periodic, and its amplitude falls very fast with the frequency of the signal. The geometry is illustrated above. Below, we show a graph of amplitude as a function of spatial frequency, for different inclinations of the small patch. This means that if you observe a high frequency signal, it didn't come from a distant source.