

CSCI 5561: Computer Vision
 Spring 2009
 Prof. Paul Schrater
 Homework #1, Due Feb. 13th

Do the problem set below for credit.

MATH/WRITING problems should be submitted in a standard electronic format (e.g. pdf, doc, ..)

PROGRAMMING problems involving Matlab code must be submitted in EXECUTABLE .m files (commands should be in order and the file should be executable).

Problem 1 (10%)

a) Show that the image of an ellipse in a plane, not necessarily one parallel to the image plane, is also an ellipse. Assume perspective projection. Here is one possible solution approach: it may help to define the ellipse as a quadratic form in homogenous coordinates:

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}^T \begin{pmatrix} \frac{1}{a^2} & 0 & 0 & 0 \\ 0 & \frac{1}{b^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = 0.$$

This gives the equation with respect to special coordinates: The ellipse is in the x-y plane. However, we can use a rigid body transformation to define a general ellipse. New

coordinates $\begin{pmatrix} x_n \\ y_n \\ z_n \\ 1 \end{pmatrix}$ are given by:

$$\begin{pmatrix} x_n \\ y_n \\ z_n \\ 1 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}.$$

Let $T = \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

The ellipse expressed in new coordinates is

$$\begin{pmatrix} x_n \\ y_n \\ z_n \\ 1 \end{pmatrix}^T T^{-T} \begin{pmatrix} \frac{1}{a^2} & 0 & 0 & 0 \\ 0 & \frac{1}{b^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} T^{-1} \begin{pmatrix} x_n \\ y_n \\ z_n \\ 1 \end{pmatrix},$$

where T^{-T} is the inverse transpose of the rigid body transformation matrix. Now a projection is a kind of transformation. Show that the projection induces a quadratic form on homogeneous image coordinates.

b) Show that the image of a line in space is a line in the image. Assume perspective projection.

Problem 2 (10%)

Do problem 2.12 in Forsyth & Ponce. If $p_A = [a \ b \ c \ -d]^T$ denotes the homogeneous coordinate vector of a *plane* P in the coordinate frame (A), what is the homogeneous coordinate vector p_B in the frame B? Hint: In homogenous coordinates, all points in the plane are perpendicular to p_A : $p_A \cdot [x \ y \ z \ 1]^T = 0$. If the points undergo a transformation, the transformed points must be perpendicular to the transformed coordinate vector.

Problem 3 (10%)

- Write a function that takes a 6-d vector of extrinsic parameters and a 5-d vector of intrinsic parameters and returns the 3x4 homogenous projection matrix.
- Write a function that computes image coordinates from a projective matrix and a matrix of image points. Make a projection matrix for the parameters:

`t = [1 1 10]';`

`R = [0.9278 -0.36514 -0.076558;`

`0.35675 0.92836 -0.10427;`

`0.10915 0.06943 0.9916];`

(this can be converted to axis-angle format using matlab's `logm()` function)

`u0 = -0.1;`

`v0 = 0;`

`sx = 0.75;`

`sy = 0.75;`

`f = 1;`

`theta = 90;`

and use this to compute the image coordinates of the cube:

`vertices = [0 0 0;`

`1 0 0;`

`1 1 0;`

`0 1 0;`

```
0 0 0;  
0 0 1;  
1 0 1;  
1 0 0;  
1 0 1;  
1 1 1;  
1 1 0;  
1 1 1;  
0 1 1;  
0 1 0;  
0 1 1;  
0 0 1]';
```

remember to add a row of ones (convert to homogenous coordinates)

Use plot3 to plot the original points, and plot() for the projected points.

Problem 4 (50 %)

The goal of this problem is to implement a linear calibration algorithm in Matlab based on the method described in section 3.2 (Forsyth). From the following image, we want to extract the intrinsic (focal length and center of image) and extrinsic (rotation and translation) parameters of the camera used to grab this image. We assume no radial distortion.

A typical way to calibrate a camera is to take a picture of a calibration object, find 2D features in the picture and derive the calibration from the 2D features and the corresponding position in 3D. In our case, we use as calibration object a cube 5m wide with a checkerboard pattern on it (see figure 1a). We search in the image (of size 640x640) for the 2D features corresponding to corners of the checkerboard. Figure 1b shows these features.

Since we know its exact size (5m), we can find the exact 3D position of each 2D feature relative to the bottom corner of the cube. This process of finding correspondences is simple but time consuming. This part of the work is done for you. Features2D.mat and Features3D.mat contain the 2D corner features and the corresponding 3D positions.

Your task is to write a Matlab function that takes these two lists as input and returns the calibration parameters as output. Your function should have the following form:

```
function [f,u0,v0,R,t] = calibrate(f2D, f3D)
```

where f is the focal length (in meters), u_0 and v_0 are the center of the image (in pixels), and $[R \ t]$ is the relative rigid transformation between the bottom corner of the cube and the camera. The size of camera CCD array is 1 inch by 1 inch and its pixels are square.

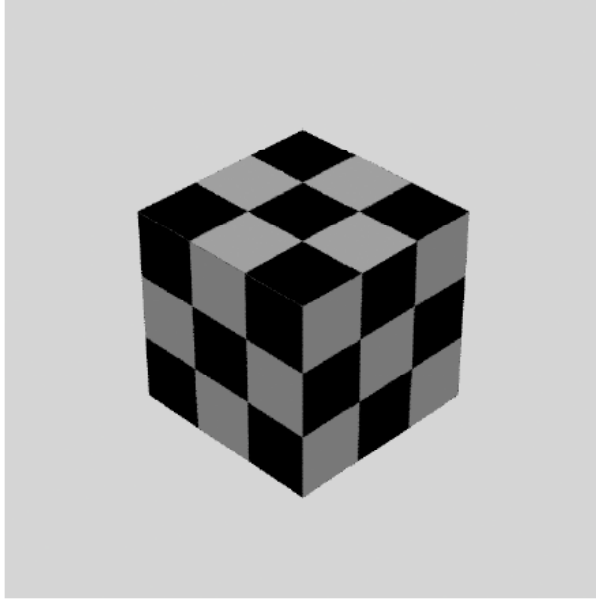


Figure 1a: Original Image

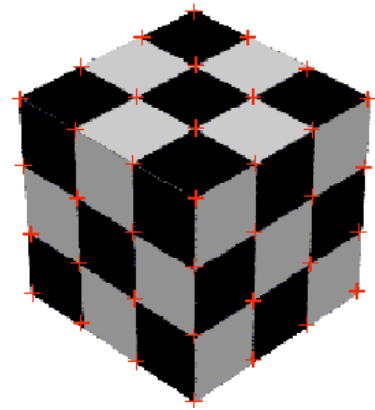


Figure 1b: Detected features

Problem 5: (10%)

Why are specularities brighter than diffuse reflection?

Problem 6: (10%)

A surface has a constant BRDF. What is the maximum possible value of this constant? Now assume that the surface is known to absorb 20% of the radiation incident on it (the rest is reflected); what is the value of the BRDF?